The Power of Stars

Despite their docile appearance in the night sky, stars are giant-size nuclear reactors that pump out enormous amounts of energy. This is most easily seen with our own Sun. Earth, except for some continuing geothermal activity at its center, gets all its energy from the Sun. And Earth only gets a very, very small part of the Sun's energy – only that part that's on a general straight-line from the Sun to us. That small percentage of the Sun's total energy output runs this entire planet.

And the Sun isn't even a big star. It's slightly in the upper half of its class, generally speaking, but there are stars out there – lots of them – that are much, much bigger. And these big stars not only produce more energy than our Sun, they produce *disproportionately* more energy.

The Dynamics of the Stellar Engine

I'm not going to get into how stars form, or what happens when they 'die' (as they all will). I'm going to be discussing what's going on in a normal-life star and how its energy is produced.

Stars are massive. Even our own Sun, in addition to being 100X the diameter of the Earth, contains 333,000 times the Earth's mass. About 73% of this mass is hydrogen, which is the fuel used in the nuclear process.

Whenever a large mass – larger that, say, the size of the ex-planet Pluto – is accumulated, this mass does its best to form itself into a sphere, because this is the most efficient shape for any particular volume, and make itself as small (dense) as possible. These are accomplished through gravitational contraction. In the case of Earth, the contraction stops when the electron repulsion of atoms and molecules balances the gravitational forces. In other words, we just can't pack it any tighter.

Now, I said I wasn't going to discuss star formation, but there's one part I can't duck. When the cloud of gas and dust that would be the Sun went though its initial gravitational collapse, about 5 billion years ago, this squeezing created heat, most especially at the center of the mass. At some point, the temperature got hot enough to where hydrogen could fuse into helium – in other words, the nuclear fire started.

I'm not going to go into detail on this fusion process, but very generally, four hydrogen atoms (one proton each) fuse into a single helium atom (two protons, two neutrons). However, this new helium atom contains about 0.7% less mass than the four hydrogen atoms that created it. This missing mass gets converted into energy by Einstein's $E=mc^2$.

Once fusion starts, we have an energy source that's trying to push the mass of the star outward, and gravitational forces are trying to contract it. At some point equilibrium is established and maintained. If gravity starts to win, this compacts and heats the core, which increases fusion activity, which in turn counteracts the gravity. And if fusion starts to win, the star and the core expand, reducing fusion, and gravity pulls it back down. This continues until the nuclear fuel – hydrogen – is used up to the point where fusion can no longer counteract gravity. In the case of the Sun, this will happen about 5 billion years from now.

The heat from this nuclear process finds its way to the surface of the Sun, where it's radiated into space in the form of "electromagnetic radiation", which we frequently, though often incorrectly, refer to a "light". But the Sun radiates more than visible light. Electromagnet radiation includes not just visible light, but also light with shorter wavelengths that visible-blue (ultra-violet, x-rays, gamma-rays) and with longer wavelengths than visible-red (infrared, radio, microwave).

At this point I'm going to have to start quantifying things, so I'll introduce you to the standard (SI) units of measure.

Units of Measure

Length and Area: The SI unit for length is the **meter** (about one yard), and for area it's the square meter (**meter^2**), which is about a square yard. I do have to note that parts of this paper refer to the **kilometer**, which is 1000 meters. It's not a unit in the SI system, but is a derived unit from the meter.

Energy: The SI unit for energy is the **joule**, and we can get this into everyday perspective. The English unit for energy is the foot-pound. One foot-pound is the energy required to lift a one-pound object one foot off the ground. If that object is about 74 pounds, the energy expended in lifting it one foot off the ground is 74 foot-pounds, which is very close to one 100 joules.

Power: This is the rate at which energy is being created or expended, and the SI unit for power is the **watt**. One watt, by definition, is one joule per second (joule/sec). This is the same 'watt' we use for light bulbs. So, if you lift a 74 pound weight a foot off the floor and put it back, and repeat this once a second, you're supplying 100 watts of power. (In putting it back down, gravity is supplying this energy, so it's also supplying 100 watts. The weight itself is moving 2 feet/second, which requires 200 watts, so it adds up.)

Mass: The SI unit for mass – which we'll be using here and there in this paper, is the **kilogram** or **kg**. Yes, it seems out-of-place since it's 'derived' from grams. Still, the powers that be, realizing that the kilogram was such a popular unit of measurement, kind of caved in and let this 'kilo' be admitted to SI. One kilogram is approximately equal to 2.2 pounds (which is a *lot* of odd-smelling tobacco).

Temperature: The SI unit of measure for temperature is the **Kelvin** (or just **K**). It's not "degrees Kelvin" – it's just "Kelvin". Since we'll be talking about stars – which mean really high temperatures – it's a pretty easy conversion to centigrade or Fahrenheit. At these values, consider Kelvin and degrees centigrade to be equal. To convert Kelvin to degrees Fahrenheit, multiply Kelvin by two if you're not fussy or by 1.8 if you are.

Scientific Notation: For really big numbers, I'll use scientific notation. For example, instead of writing "1,000,000" for one million, I'll write "1×10^6". I don't describe scientific notation in this paper. You're on your own for this one.

Luminosity: This is not an SI unit. For stars, we refer to luminosity as the total power output of the star. Since this luminosity is power, its unit of measure is the **watt**.

Next, I'm going to use these units – and an equation – to calculate the luminosity of the Sun.

The Luminosity of the Sun

At this point, we can't duck using some algebra. I'm going to show how the luminosity of a star is calculated, and then relate this to the Sun. Now, I'm not going to ask you to actually plug numbers into this equation, but you do need to understand what it means so you can relate luminosity to that which contributes to it.

The equation for calculating luminosity is:

$\mathbf{L = 4\pi r^2 \sigma T^4}$ watts

Where:
L = the star's luminosity (in watts)
π (Pi) you'll recognize from high school. The value I'll be using in computations is 3.14159.
r is the radius of the star in meters.
σ (Sigma – the Stefan-Boltzmann constant) is kind of strange. You can use a rounded value of 5.6704×10^{-8}. Its units are even weirder: watts/meter²/K⁴, (watts per square meter per Kelvin to the 4th power). But don't worry about that. It's just a constant – a number.
T is surface temperature of the star in Kelvins.

There is a subtle but key point here. If you're interested in the power output of a star, all you need know is its radius 'r' and its surface temperature 'T'. All the other things on the right side of that equation are constants.

Now, the radius of the Sun is 6.955×10^8 meters, and it has a surface temperature of 5778K (these values from Wikipedia). If you want, plug these into the above equation. In any case, the Sun's luminosity is about (depending on your rounding errors) 3.846×10^{26} watts.

Before leaving this equation, there's something else I want to show you. We can break up the right side of the equation into two parts, thus:

$\mathbf{L = [4\pi r^2][\sigma T^4]}$

But $\mathbf{4\pi r^2}$ is nothing more than the surface area $\mathbf{A=4\pi r^2}$ of a sphere or, in this case, the surface area of the Sun. So we can also write this equation:

$\mathbf{L = A\sigma T^4}$ watts

So, luminosity depends on the surface area, and the temperature of that surface. And you should note this: If you double the surface area, you double the luminosity of the star. But if you double the temperature, you increase the luminosity ***by a factor of 16*** (since $2^4=16$). It's the surface temperature of the star that's the key contributor to its luminosity.

"Unit-Area" Luminosity

We're just about done with the equation, but before we leave, let's look at one final thing.

Luminosity **L** is the total luminosity (watts) of the star. If we divide both sides of the equation by the star's surface area **A** we get unit-area luminosity:

$\mathbf{L/A = \sigma T^4}$ watts/meter^2

This is the power produced by every square meter of the star. I'll do the calculation for you for the Sun:

L/A = 63,200,984 watts/meter^2 for the Sun.

Now, L/A actually has several formal names, one of the most common being "radiant flux" and the symbol that I've found most commonly used for L/A is **j***., which I find kind of confusing since **j** usually implies energy. But the most important point is, every square meter of the surface of the Sun produces the power of a 63-megawatt nuclear power plant.

We are now, at last, done with the equation.

Distance: The Light Year

How far away a star is (from anything) has absolutely nothing to do with its power. But in discussing stars other than the Sun, it helps to know how far away the star is, so I just can't duck this topic.

Astronomical distances, beyond our solar system, are measured in "light-years". Now, you'd think the 'years' part implies a unit of time, but it doesn't. The light-year is the distance light travels in one year in a vacuum. It's abbreviated "LY", or alternatively "ly", and equals approximately 6 trillion miles, or 10 trillion kilometers. (One trillion = 1×10^{12}).

By comparison, our Sun is “only” 93 million miles, or 150 million kilometers. This, by comparison, is about equal to 8.5 light-seconds.

Now “real” astronomers have an alternate unit of measure for distance, called the “parsec”, abbreviated **pc**. I’m not going into detail, rhyme, reason, or math, but one parsec equals approximately 3.26 light-years (or, 1 pc = 3.26 ly). We mortals almost always use light-years.

Let’s Take a Break

We’ve talked about how the star’s nuclear engine works, and the forces that keep its size what it is. We’ve looked at the scientific units of measure, and related them to everyday-life quantities. And we’ve looked at the Sun, and what the pertinent solar measurements are.

The problem is – these numbers are so *big!*

For example, I told you the radius of the sun is 6.955×10^8 meters. Now, if there was a typo here, and I mistakenly told you the radius was 6.955×10^9 meters, would you notice the difference? Would it be obvious? Can you picture that kind of distance?

Probably not. So, when I introduce new stars to you – bigger than the Sun – using this kind of measurement would do you no good at all. We need a new kind of measurement that you can relate to.

And we have something – the Sun itself. This is something you see every day, that you can relate to. So, we’ll take advantage of that and use it as our new unit of measurement.

Using the Sun as a Baseline Measurement

First, we need a new symbol – “$\odot$” – to represent the Sun, and here’s the notation we’ll use for our new baseline.

$\mathbf{R_\odot}$ refers to the radius of the Sun. Thus $\mathbf{78R_\odot}$ reads as “78 solar radii” (which you can, of course, calculate to the meter if you want). $1R_\odot = 6.955\times10^8$ meters.

$\mathbf{M_\odot}$ is used for the mass of the Sun. $\mathbf{10M_\odot}$ means “ten solar masses”. $1M_\odot = 2\times10^{30}$ kilograms, approximately.

$\mathbf{L_\odot}$ is for luminosity. $\mathbf{100L_\odot}$ means “100 times as luminous as the Sun”. $1L_\odot = 3.846 \times 10^{26}$ watts.

$\mathbf{T_\odot}$ refers to the surface temperature “T” of the Sun and is not commonly quoted in star catalogs. This is because, although this number is big, it’s still manageable as-is.

However, I'm going to use it here because it will save you some calculator time.
$1T_\odot$ = 5778K. Note the 'K' stands for Kelvin and *not* for 'thousands'.

Now, I certainly didn't invent this clever way of putting the characteristics of stars into a perspective relative to the Sun. Using the Sun itself as a unit of measurement has been used by astronomers and star catalogs for a long time.

Before we look at and compare some stars, you probably already know you can look up the data on the internet. Because we're using "Solar" units, there's another equation you might want to know:

$$\mathbf{L_\odot = R_\odot^{2} \bullet T_\odot^{4}}$$

This says, "The solar luminosity of a star is its solar radius, squared, times the solar surface temperature, raised to the fourth power."

Now be *careful!* If you actually want to use this equation, the "T" value is *solar* surface temperature – yet, almost all star data you'll see is the star's *actual* surface temperature, and this value won't work in this equation. You will probably have to do your own conversion by dividing the actual value given by 5778, which is the Sun's surface temperature. If you do that, you'll fine.

Now let's take a look at some of our stellar neighbors.

Proxima Centauri

At only 4.3 light-years away, this star is closest to us (excluding, of course, the Sun). Despite this distinction, it's pretty much a wimp.

$R_\odot = 0.145$; $T_\odot = 0.526$; $L_\odot = 0.0017$

Why is its luminosity only 0.2% that of the Sun? Well, it's radius on only 15% the Sun (not much of a surface area), and worse, its temperature is only about half. Remember that surface temperature – being raised to the fourth power – is a major contributor to luminosity.

Sirius

Sirius is also a relativity close star (8.6 light years), and it's pretty big, and it's bright.

$R_\odot = 1.1711$; $T_\odot = 1.72$; $L_\odot = 25.4$

At first glance, this doesn't seem to make sense. It's less than twice the Sun's radius, and its surface temperature is less than twice the Sun's – but it's 25 times as luminous! How can this be? You have to remember radii are squared, and surface temps are raised to the fourth power. A star that has twice the Sun's radius and twice the Sun's temp would equate out to $2^2 \bullet 2^4 = 4 \bullet 16 = 64$ times as luminous. The $R_\odot$ and $T_\odot$ values may go up slowly, but luminosity does not!

Here's the reason – as the radius increases, that just that much more mass squeezing down on the nuclear core. To keep the star in balance – to keep it from collapsing – the nuclear activity has to step up its pace, generating a *lot* more heat. (And, by the way, using up its nuclear fuel at a much-increased rate.)

Vega

This my very favorite star. It's not as bright as Sirius, but that's only because it's farther away at 25.3 light years.

$R_\odot = 2.26$; $T_\odot = 1.66$; $L_\odot = 37.0$

There's a curiosity here. Do you see it? Vega is significantly larger than Sirius – yet its surface temperature is actually less! This is because, even though our luminosity calculations work perfectly for a specific star, it generally doesn't work to take these values as predictive of other stars. All stars are *not* created equal, and the method by which they set up their nuclear reactors differs. This is not pure mathematics.

Summary

At this point, you should have a pretty good idea of how powerful stars are, what makes them that powerful, and how to use and interpret star data that you can find on the Internet. I hope this paper was as much fun for you to read as it was for me to write.

Credits

Special thanks go to:

- **Prof. Alex Filippenko**, UC Berkeley, and his Teaching Company DVD series "Understanding the Universe: An Introduction to Astronomy"
- **Prof. Neil deGrasse Tyson**, Director of the Hayden Planetarium, and his Teaching Company DVD series "My Favorite Universe"
- **Wikipedia.com**, for more reference material than can be listed.